Simultaneous Inpainting of Image Structure and Texture with PDE

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Abstract

We propose to modify a partial differential equation model for image inpainting by introducing an additional term representing texture information into the PDE so that both image structure and texture are propagated into the missing area. The objective is to solve the problem of lacking texture information in most PDE-based methods. The proposed method does not need to decompose the image into different layers. Structural and textural details in the damaged region are generated simultaneously. Experimental results show that the method can produce more naturally looking inpainted image patches in a damaged texture region.

1. Introduction

Image inpainting has attracted much research attention since the late 1990s, which aims to repair damaged images in an indiscernible way. Traditional image restoration attempts to reduce noise or restore image degradation such as motion effects or blurring in order to obtain an optimal (in some sense) estimate of the original. Image inpainting, on the other hand, is to imitate human professionals in repairing damaged images using some mathematical methods and computer algorithms to fill in or regenerate the damaged part of the image. The objective is to produce visually satisfactory or acceptable results, rather than a good estimate of the original since there is simply no original. Inpainting of digital images has found applications in many areas including restoration of historical photographs, filling in or removing chosen areas in images, removing visible watermarking, and repairing missing blocks of JPEG images due to transmission over poor channel, etc.

Some typical techniques use partial differential equations (PDE) such as the approaches proposed by Bertalmio *et al.* [1,2]. They established a mathematical model of image inpainting by borrowing ideas from classic fluid dynamics to treat image problems. By iteratively solving the numerical representation of a PDE, they managed to smoothly propagate information of gray values from surrounding areas into the region $\mathbf{\Omega}$ to be inpainted along isophotes. The inpainting process is terminated until the gray values in the computation domain reach a stable state. Guided by the connectivity principle of human visual perception, Chan *et al.* [3] introduced a third-order PDE model for inpainting non-texture images. This model actually represents an anisotropic diffusion process. To satisfy the human visual requirements, the intensity of diffusion is related to the curvature. Shi *et al.* proposed an adaptive inpainting algorithm that is equivalent to nonlinear interpolation [4]. The repairing procedure checks the surrounding information of a damaged pixel and determines the size of the reference window that can be used to compute an interpolated color, but this method cannot produce satisfactory results when the repaired pixels are close to edges.

The major problems with the PDE models are the lack of ability for reconstructing textures in the damaged regions. Therefore textures have to be generated in a separate process. The method proposed in [5] decomposes image into structure and texture layers, and reconstructs each layer respectively with a PDE-based algorithm [1] and a texture synthesis algorithm [6]. The retouched image is finally obtained by integrating these two sub-images. We have used a texture replication technique [7] to amend the deficiency in a heat transfer modeling of image inpainting in which structure information is repaired through iteration, while the texture is obtained by subtracting a low-passed version of the neighborhood of damaged region from the original, and added to the inpainted structure. All these methods use non-PDE methods to treat textures, the image decomposition may be rather complicated therefore time-consuming, and the results are often unsatisfactory.

In this work, we propose to modify the typical PDE-based inpainting method by adding an additional term representing texture information so that both structural and textural information can be propagated into the target region simultaneously. Compared with the layer-based methods, this technique eliminates the need for image decomposition, and the algorithm is more efficient. We will describe the modified PDE model that propagates both the structural and textural information simultaneously, and give its finite difference implementation in Section 2. Experimental results are presented in Section 3 to show effectiveness of the method. Section 4 concludes the paper.

2. A PDE-based method capable of propagating texture information

Let $\mathbf{I}$ be a damaged image, with $I(x, y)$ being gray value of pixel (x, y). We start from the PDE-based inpainting model of [1-2]. Let Ω be the region to be inpainted in $\mathbf{I}$, and $\partial\Omega$ the boundary of Ω. The Laplacian $\Delta I(x, y)$ is used as a smoothness measure of the image. In a smooth region $\Delta I(x, y) \approx 0$, while in a fluctuating region $\Delta I(x, y)$ is large. Imitating the practice of a traditional art professional in the manual repairing, details in the damaged region may be *created* by extending the known information in the surrounding areas into the damaged region along isophote directions. The field of isophote is defined as:

$$\nabla^{\perp} I(x,y) = \left(-\frac{\partial}{\partial y}\mathbf{i} + \frac{\partial}{\partial x}\mathbf{j} \right) I(x,y) \tag{1}$$

Clearly, changes in image gray values are minimal along the isophote directions. Having finished the inpainting process, $\nabla^{\perp} I(x, y)$ should be normal to the gradient of the smoothness $\Delta I(x, y)$:

$$\nabla \left[\Delta I(x,y) \right] \cdot \nabla^{\perp} I(x,y) = 0 \tag{2}$$

The scalar product in the above equation indicates projection of the smoothness change onto the direction of isophote. Let the projection value be equal to the change of image gray values with respect to time, we can obtain the following partial differential equation:

$$\frac{\partial}{\partial t} I(x,y) = \nabla\left[\Delta I(x,y)\right] \cdot \nabla^{\perp} I(x,y), \qquad \forall (x,y) \in \Omega \quad (3)$$

By discretizing (3), we obtain an iteration algorithm for inpainting. As this model propagates information of gray values along isophote directions, it can effectively repair the structure without serious blurring on edges. In case there are rich textures in the damaged region, however, using the above method will not produce satisfactory results because it has not taken the periodicity of texture into account.

In order to improve the inpainting effect, the texture feature must be included in the model so that the algorithm can generate coherent texture in the inpainted region while repairing the structure. Consider the situation of Figure 1. We can make use of the periodicity of texture to propagate the texture information along directions of the texture and the perpendicular direction respectively while propagating the structure information along isophote directions.

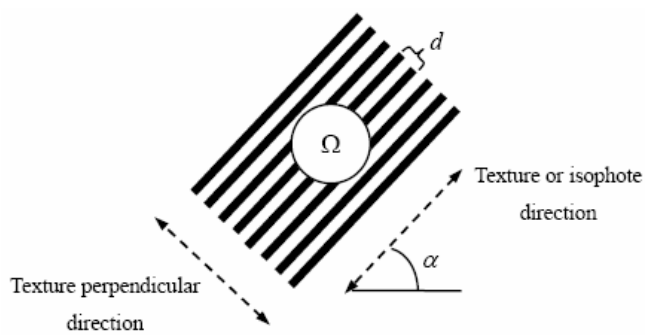

Figure 1. *Propagating direction of image information*

Let $\alpha \in [0, \pi]$ be the angle between the texture direction and the horizontal line, and d the scale of texture periodicity. We introduce the texture ingredient into (3), resulting in the following modified PDE for image inpainting:

$$\frac{\partial}{\partial t} I(x,y) = A\nabla\left[\Delta I(x,y)\right] \cdot \nabla^{\perp} I(x,y) + B\Delta I(x,y;\alpha,d), \quad \forall (x,y) \in \Omega \quad (4)$$

where A and B are weights for structure and texture respectively, A, $B \in [0, 1]$, and $A + B = 1$. If $B = 0$, (4) is reduced to (3), meaning that only the structure is inpainted. The generalized Laplacian $\Delta I(x, y; \alpha, d)$ contains texture components:

$$\Delta I(x,y;\alpha,d) = \left(\frac{\partial^2}{\partial \xi^2} + \frac{\partial^2}{\partial \eta^2}\right) I(x,y;d) \quad (5)$$

$$\xi = x\cos\alpha + y\sin\alpha \quad (6)$$

$$\eta = -x\sin\alpha + y\cos\alpha \quad (7)$$

where ξ and η corresponds to the texture direction and its perpendicular direction respectively. The discrete form of the second-order partial difference with respect to ξ and η are:

$$\frac{\partial^2 I(i,j;d)}{\partial \xi^2} = I(i+d_y, j-d_x) + I(i-d_y, j+d_x) - 2\cdot I(i,j) \quad (8)$$

$$\frac{\partial^2 I(i,j;d)}{\partial \eta^2} = I(i+d_x, j+d_y) + I(i-d_x, j-d_y) - 2\cdot I(i,j) \quad (9)$$

where

$$d_x = \mathrm{round}(d\cos\alpha), \quad d_y = \mathrm{round}(d\sin\alpha) \quad (10)$$

Since the gray values show repetition along the texture and its perpendicular direction, (5) reflects gray value difference between the damaged pixels with intervals related to the texture periodicity. Therefore the model represented by (4) can simultaneously propagate structure and texture information in the course of iteration.

The structure-texture integrated PDE model for image inpainting as given in (4)~(10) requires the following input parameters: weights A and B for the structure and texture information, texture direction α; texture periodicity d, and a mask that identified the region Ω to be repaired. Inpainting is carried out within region Ω, and the rest of the image is unaltered. To use an iterative procedure to solve the PDE (4) numerically, we define discrete forms of the pixel gray values as follows:

$$I^{(n+1)}(i,j) = I^{(n)}(i,j) + \Delta t\, I_1^{(n)}(i,j) \qquad n = 1,2,\ldots,T \quad \forall (i,j) \in \Omega \quad (11)$$

$$I_1^{(n)}(i,j) = A\, I_s^{(n)}(i,j) + B\, I_t^{(n)}(i,j) \quad (12)$$

The superscript (n) is step number of iteration. $I^{(n)}(i, j)$ is the updated result after n steps of iteration. Δt is updating step interval. $I_1^{(n)}(i, j)$ is the increment of the gray value at (i, j) and step n. The structure component is

$$I_s^{(n)}(i,j) = \nabla\left[\Delta I^{(n)}(i,j)\right] \bullet \nabla^{\perp} I^{(n)}(i,j) \quad (13)$$

Its discrete form can be found in [1]. The texture component $I_t^{(n)}(i, j)$ is $\Delta I(i, j; \alpha, d)$, with the discrete forms given in (5)-(10). It is clear that the larger the inpainted region, the more iteration steps T are needed. T may be pre-determined on a trial-and-error basis, or the iteration may terminate when $|I_1^{(n)}(i, j) - 0|$ is less than a given small positive number ε.

3. Experimental Results

Experiments were carried out on a group of 512×512 images. An example is the test image Barbara with rich texture as shown in Figure 2. Suppose the region to be inpainted is located in a texture-rich area marked with white blocks in the figure. The parameters used in the experiment are: weight of texture information $B = 0.5$, texture periodicity $d = 4$ (measured from the test image), the number of iteration steps T is set to 200.

The inpainted regions are magnified to make the inpainted details clear. Figure 2(a) and (b) show the damaged images. The inpainted results are presented in (c) and (d). The texture directions in the two regions are about $4\pi/9$ and $\pi/6$ respectively. As a comparison, the damaged image is also inpainted using the method in [1], with 3000 iterations. The results are given in (e) and (f). It is observed that the method proposed in the present work produces better visual appearance. Both the structure and texture

are repaired, while the method of [1] does not provide satisfactory results.

The proposed method is efficient since the structure and texture are propagated simultaneously. We executed our Matlab codes on a computer with a PIII 1.5GHz processor and 256MB memory under Windows XP. The inpainting process for Figure 2(a) and (b) took less than 63 and 68 seconds respectively, while for the method of [1], it took more than 10 minutes.

4. Conclusions

In order to solve the problem of lacking texture information in the inpainted region produced by PDE-based methods, we have introduced a texture component in a partial differential equation. The added term reflects periodicity along the texture and its perpendicular direction. The proposed modification to the inpainting model originally described in [1] leads to a new approach that propagates both structure and texture simultaneously into the damaged area, and is computationally efficient. Satisfactory results are obtained in inpainting damaged areas with texture.

Parameters such as the texture direction, periodicity and weights need to be pre-determined at the current stage. Further investigation is needed to find ways for automatic, or semi-automatic, determination of these parameters. One limitation of the present method is that the texture must have a dominant direction. Further improvement to be made is therefore to make the model more general so that irregular textures can effectively be treated.

References

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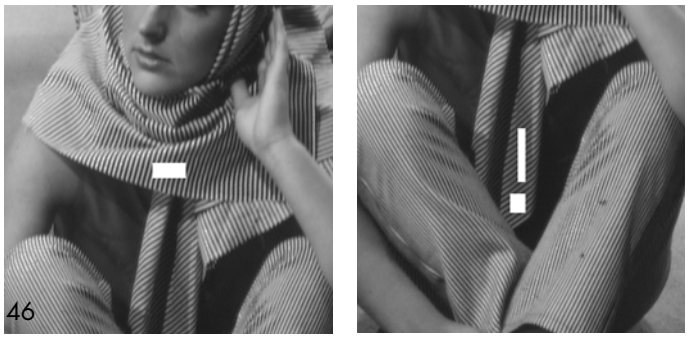

(a) Damaged image No.1 (b) Damaged image No.2

(c) Inpainted result No.1 (d) Inpainted result No.2

(e) Inpainted result No.1 based on [1] (f) Inpainted result No.2 based on [1]

***Figure 2**. Experimental results*

Author Biography

Chuan Qin received his BS and MS in Electronic Engineering from the Heifei University of Technology at 2002 and 2005 respectively .Now he pursues his PhD in Signal and Information Processing from Shanghai University. His work has focused on image inpainting based on PDE and information hiding.